23 Useful Formulae

23.1 Inverse Square Law

D_1 = original distance from source

D_2 = new distance from source

R_1 = original radiation field

R_2 = new radiation field

$$R_2 = R_1 \frac{(D_1)^2}{(D_2)^2}$$

e.g., if a source is 10 mrem/h at 5 feet, then at 13 feet it is

$$10 \times \frac{5^2}{13^2}\ mrem/h = 1.5\ mrem/h.$$

23.2 Air Sample Calculations

Particulates (complete calculation):

Sample collection time = T minutes

Collection flow rate = F m^3/minute (usually 0.7)

Counter efficiency = E%

Background count rate = A_1 CPM

Count rate with sample in tray = A_2 CPM

$$DPM/m^3 = \frac{(A_2 - A_1)\,100}{E} \times \frac{1}{TF}$$

23.3 Radioactive Decay

The following symbols will be used in this section:

N_0 = number of unstable nuclei at some original time

N = number of unstable nuclei remaining after a time interval t

I_0 = intensity of radiation at some original time

I = intensity of radiation after a time interval t

A_0 = activity of sample at some original time

A = activity remaining after a time interval t

λ = decay constant for particular radioactive element

e = base of natural logarithms, 2.718

$T½$ = half-life

$n = t/T_{1/2}$ = number of half-lives

(1) $N = N_0e^{-\lambda t}$ or $N = N_0e^{-0.693t/T_{1/2}}$

(2) $A = A_0e^{-\lambda t}$ or $A = A_0e^{-0.693t/T_{1/2}}$

(3) $I = I_0e^{-\lambda t}$ or $I = I_0e^{-0.693t/T_{1/2}}$

(4) $N = N_02^{-n}$ or $N/N_0 = 1/2^n$

23.4 Decay Constant

$$\lambda = 0.693/T_{1/2}$$

23.5 Specific Activity

Specific activity = $\lambda N = 0.693\ N/T_{1/2}$ = dis/sec/g

where $T_{1/2}$ = half-life in seconds

N = number of atoms per gram

$$\textit{Specific activity} = \lambda N (3.7 x 10^{10}) = \frac{N x 1.873 x 10^{-11}}{T_{1/2}} = Ci/g$$

23.6 Radiation Absorption

Alpha Particle Range

For E < 4 Mev:

$$R_\alpha = 0.56E$$

For 4 < E < 8 MeV

$$R_\alpha = 1.24E - 2.62$$

where R_α = range in cm of air at 1 atm and 15°C
E = energy in Mev

Beta Particle Range

For $0.01 \le E \le 2.5$ Mev

$$R = 412E^{1.265 - 0.0954 \ln E},$$

$$\ln E = 6.63 - 3.2376(10.2146 - \ln R)^{1/2},$$

where R = range in mg/cm^2,

E = maximum energy in Mev.

For $E \ge 2.5$ MeV

$$R = 530E - 106,$$

Where R and E are the same as above.

Sargent's Rule (E > 0.8 Mev)

$$R = 0.0526E - 0.094,$$

where R = range in g/cm^2,
E = maximum energy in Mev.

Feather's Rule (E > 0.6 Mev)

$$R = 0.542 - 0.133,$$

where R and E are the same as for Sargent's Rule.

Gamma Ray Absorption

The following symbols will be used in this section

I_0 = original radiation exposure rate
I = attenuated radiation exposure rate
μ = linear absorption coefficient $(cm^{-1}) = \frac{0.693}{X_{1/2}}$
μ/ρ = mass absorption coefficient (cm^2/g)
ρ = absorber density (g/cm^3)
x = absorber thickness (cm)
$x_{1/2}$ = half-value layer of absorber (cm)
e = base of natural logarithms (2.718...)
b = "build-up" factor

For monoenergetic or monochromatic narrow-beam radiation,

$$I = I_0 e^{-\mu x}$$

or

$$I = I_0 e^{-(\mu/\rho)(\rho)(x)} ;$$

for monoenergetic or monochromatic wide-beam radiation,

$$I = bI_0 e^{-\mu x}.$$

Neutron Absorption (for a collimated beam of monoenergetic neutrons)

$$I = I_0 e^{-\sigma N x},$$

where I_0 = initial neutron intensities,

I = final neutron intensities,

N = number of atoms per cc in the absorber,

σ = cross section (cm^2)

x = thickness of absorber (cm),

e = base of the natural logarithm (2.718...).

Since this equation is only an approximation of neutron attenuation, average neutron energies can be used for determining the value of σ. The equation is not accurate enough to justify the use of neutron build up factors.

Approximate Range-Energy Relation for Protons

$$R = (E/9.3)^{1.8} ,$$

where E = energy in Mev (few Mev to 200 Mev),

R = range in meters in air.

23.7 Beta Particle Counting

Self-Absorption

$$\frac{R_0}{R} = \frac{1}{mx} (-e^{-mx}),$$

$$\frac{R_0}{R} = \frac{1}{mx}\left(-e^{-mx}\right),$$

where R_0 = measured counting rate,

R = true counting rate

x = sample thickness (mg/cm^2)

m = absorption coefficient (cm^2/mg)

Resolving-Time Determination

$$\tau = \frac{R_1 + R_2 - R_{12}}{2(R_1 R_2)},$$

where τ = resolving time in seconds,

R_1 = counting rate, source 1(c/s)

R_2 = counting rate, source 2 (c/s)

R_{12} = counting rate, combined sources 1 and 2 (c/s)

Resolving-Time Correction

$$R = \frac{R_0}{1 - R_0\tau}.$$

where R = true counting rate (c/s),

R_0 = observed counting rate (c/s),

τ = resolving time in seconds.

23.8 Calibration Procedure

The equations in this section are applicable to dose in air from gamma emitters.

Exposure Rate from a Point Source

$$I_\gamma = 0.156nE(10^5\mu_a) \ ,$$

where I_γ = mr/hr at 1 meter per mCi,

n = gamma quanta per disintegration,

E = energy of gamma quanta in Mev,

μ_a = energy absorption coefficient for gamma in air (STP) in cm^{-1}.

The equation assumes that one ion pair in air causes an average energy expenditure of 32.7 eV.

Approximate Exposure Rate from Any Gamma Point Source

$$r/hr \ at \ 1 \ foot = 6CEn \ ,$$

or

$$mr/hr/mCi \ at \ 1 \ meter = 0.5 \ nE \ ,$$

where C = number of curies,

E = gamma-ray energy in Mev,

n = gamma quanta per disintegration.

Exposure Rate from Any Gamma Point Source

$$mr/hr = nI_\gamma/s^2 \ ,$$

where n = number of millicuries,

I_γ = mr/hr at 1 meter per mCi,

s = distance in meters.

23.9 Internal Radiation Dosage

Biological Half-Life

$$T_b = \frac{0.693}{\lambda_b} ,$$

where λ_b = biological rate of elimination constant,

T_b = biological half-life

Effective Half-Life

$$T_{eff} = \frac{(T_{1/2})(T_b)}{T_{1/2} + T_b} ,$$

where T_{eff} = effective half life,

$T_{1/2}$ = radioactive (physical) half-life

T_b = biological half life.

Beta-Emitter Dose

$$D = 88 E T_{eff} C (1 - e^{-\lambda_{eff} t}) ,$$

where D = dose (reps),

E = average energy of beta particle (Mev)

C = Ci/g of radioisotope in tissue

λ_{eff} = effective decay constant (days^{-1})

t = time (days).

23.10 Decontamination Factor

$$D.F. = \frac{Initial\ Activity}{Final\ Activity}$$