Calculation and Re-Calculation of 60-Minute Sigma Theta and Stability

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Abstract

While ANSI/ANS 3.11-2000 does not specify a method for computing σ_θ, it suggests one-pass methods in Appendix E, including the Yamartino method. For the suggested methods, it also recommends deriving the hourly value of σ_θ by computing the Root Sum Squared (RSS) of the 10 or 15-minute averages in order to minimize inflation of the 60-minute σ_θ due to the effects of plume meander.

This inflation effect was observed while developing a utility to compute missing 60-minute σ_θ's from partial data or to re-compute this value after portions of the underlying data had been edited. Re-computed σ_θ's generally are lower than the original values, frequently reducing the associated stability class to a more stable value.

This presentation reviews σ_θ computation, then details the observed differences between 60-minute σ_θ's computed from the original sensor data using the one-pass Yamartino method and values computed by RSS-ing the 15-minute values. One year of actual observations and re-computations are analyzed with respect to differing wind conditions. Lastly, the differences in reported stability class for the two methods are presented.

Introduction

The standard deviation of any continuous variable (X) is defined as:

$$\sigma_X^2 = \frac{1}{N}\sum_{i=1}^{N}(X_i - \mu)^2$$ where X is a set of samples and μ is the mean of the X values.

A little mathematics produces the equivalent expression:

$$\sigma_X^2 = \frac{1}{N}\sum_{i=1}^{N} X_i^2 - \left(\frac{1}{N}\sum_{i=1}^{N} X_i\right)^2$$

Thus the standard deviation may be computed in a "single pass" through a number of samples, accumulating the sum of the samples, the sum of the squares, and the count of the number of samples.

But wind direction is not continuous; there is a discontinuity between 360 ° and 0 °. The analogous method of computing the standard deviation of the wind direction (sigma theta) would require two passes through the data; one to compute the average wind direction and a second pass to determine the standard deviation. This would require keeping one hours worth of wind direction measurements (perhaps at one second intervals or less) in a datalogger.

Two decades ago, several methods were evaluated to determine the best one pass method for computing sigma theta (that would more closely approximate the two pass results). These methods were based on accumulating the average sine and cosine of the wind direction (some methods used additional quantities)

Historical work

Below are brief descriptions of some of the influential papers of the period.

Yamartino

Yamartino developed an estimator based on the assumption that the distribution of wind direction was isotropic, and selected an interpolation function based on its performance against Monte-Carlo simulations also based on this assumption. Yamartino benchmarked this method against the Ackerman-Verrall and Williams methods (which used more complicated estimators) and showed that the Yamartino method performed better for high values of sigma theta compared to the others.

The Yamartino method computes the average sine of wind direction and the average cosine of wind direction. From these two quantities it computes epsilon (the sine of sigma theta). Sigma theta is then computed as the arcsine of epsilon, multiplied by an interpolation factor to correct for large values of sigma theta.

$$S = \frac{1}{N} \sum_{i=1}^{N} \sin \theta_i \qquad C = \frac{1}{N} \sum_{i=1}^{N} \cos \theta_i \qquad \varepsilon = \sqrt{1-(S^2 + C^2)}$$

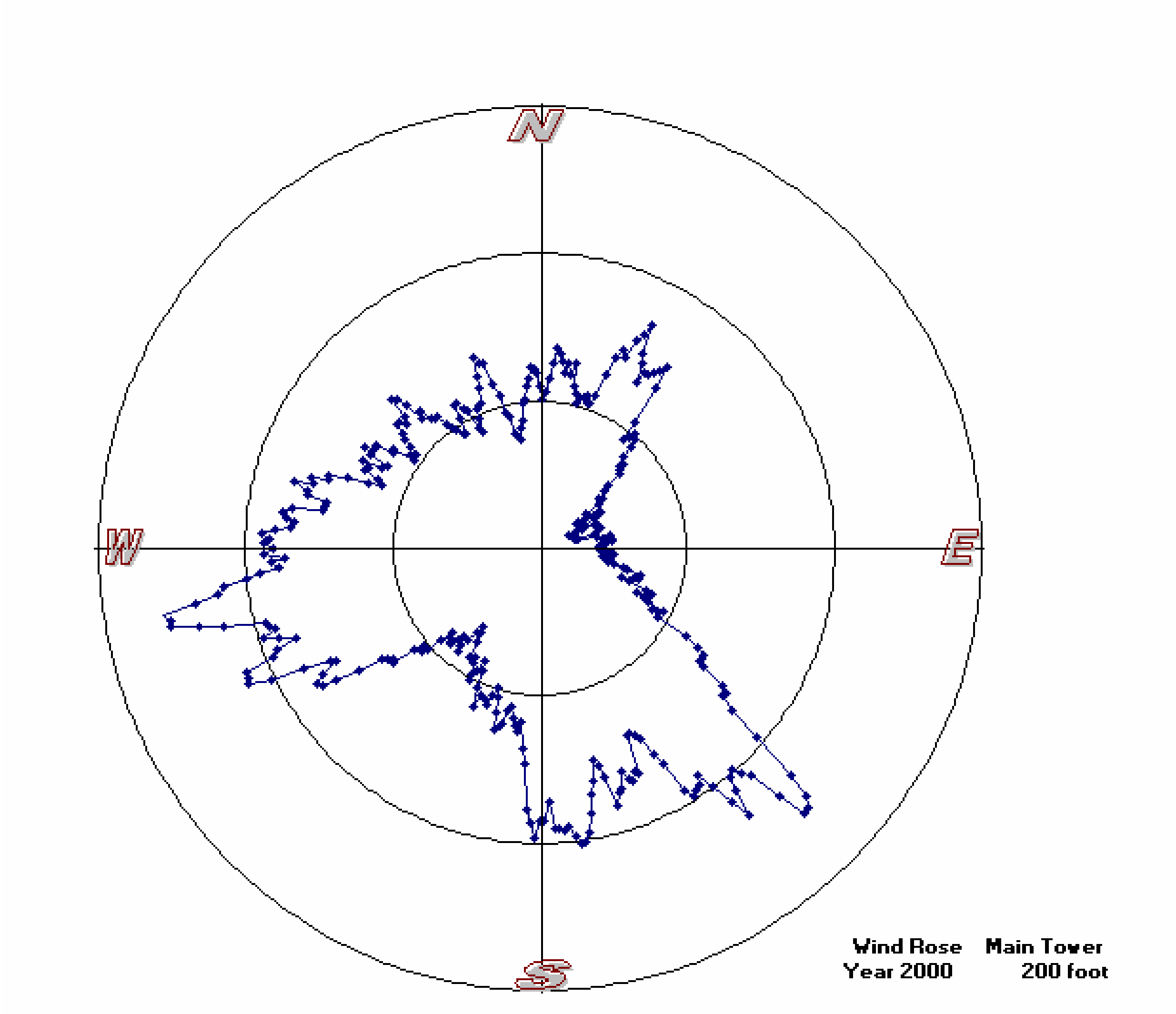
$$\sigma_\theta = \text{arcsine}(\varepsilon) \left[1 + \left(\frac{2}{\sqrt{3}} - 1 \right) \varepsilon^3 \right]$$

Turner

Turner compared the three methods (Ackerman-Verrall, Williams and Yamartino) against actual data, eight hours of .1 second sample with relatively low sigma theta values ($<22^{\circ}$). The three methods were comparable. The three methods were tested against additional simulated data and again the Yamartino method was preferred.

Fisher

Fisher challenged some of the assumptions used to justify the Yamartino, Ackerman and Verrall and Williams methods, arguing that they were based on the assumption of isotropic or simply modeled wind sources with symmetric distributions about the mean, not likely to be found in nature. He argued for more extensive computations within the capabilities of existing minicomputers of the time, but not within the capability of a datalogger.



An example of a real long term distribution of wind direction

Yamartino Method Restriction

The Yamartino method thus came to be a standard embedded in dataloggers. Experience with real data showed that it can produce inflated sigma theta values.

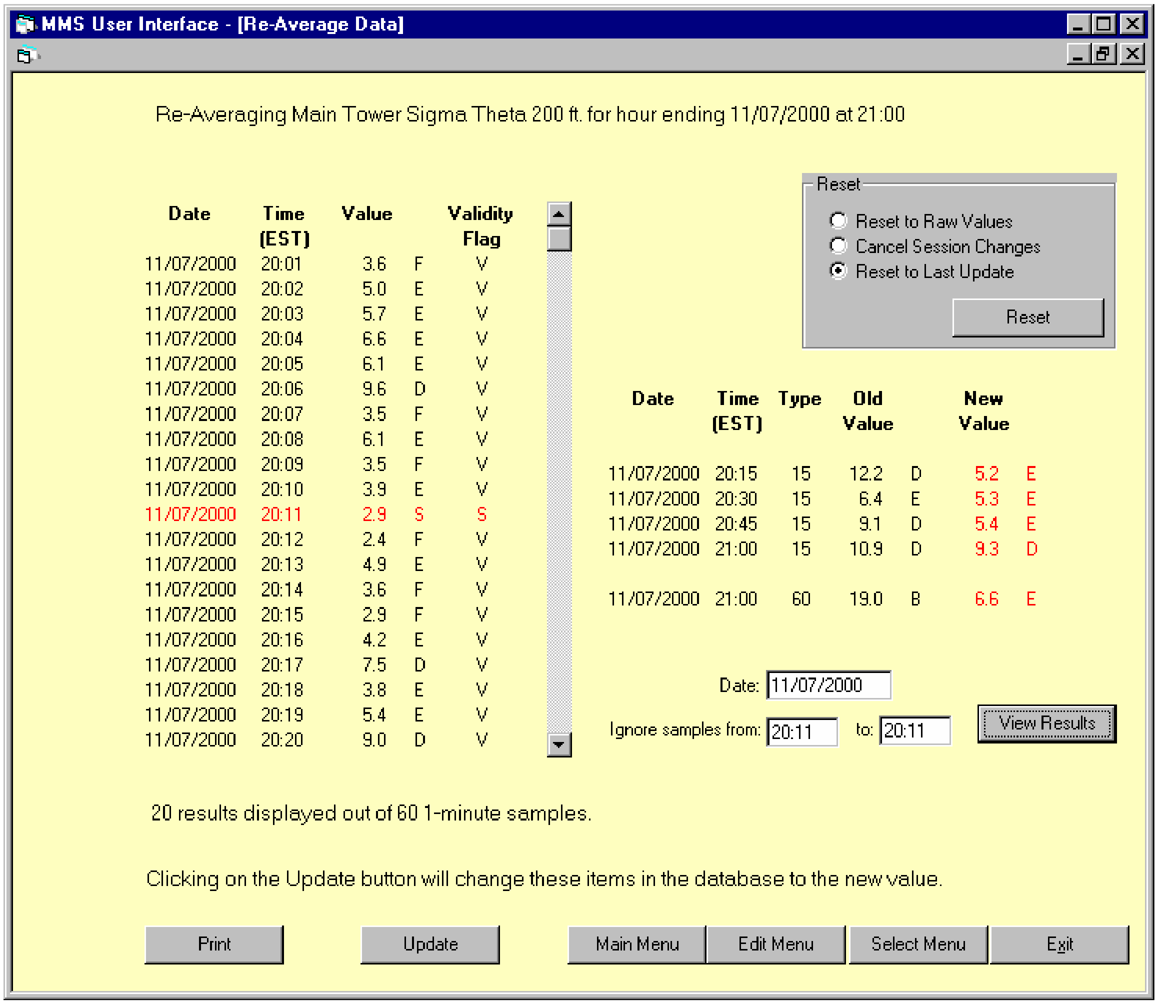
ANSI/ANS-3.11-2000 contains the following advice:

"Note that calculations of sigma theta values for a one-hour sampling period through the use of either of the algorithms presented above [Yukihiro and Yamartino] could be inflated by the contributions from long-period oscillations associated with light wind speed conditions (e.g., plume meander). To minimize the effects of plume meander … four 15-minute $\boldsymbol{\sigma_\theta}$ values should be combined to represent an hourly sampling period as follows:

$$\sigma_\theta\ (hr) = \sqrt{\frac{(\sigma_{\theta 1})+(\sigma_{\theta 2})+(\sigma_{\theta 3})+(\sigma_{\theta 4})}{4}}$$ "

Example of Problem with 60 Minute Results

How is the 60-minute sigma theta computed from the wind samples different from the sigma theta computed from the 15-minute sigma thetas? This became an issue while developing a procedure to re-compute the sigma theta when some of the underlying data was edited or to compute a 60-minute sigma theta when it is missing.



The recomputed sigma thetas were always lower than the original sigma thetas. This triggered an analysis of sigma theta calculation.

Pasquil Stability Class

The analysis below describes changes in both sigma theta and stability class, which is computed from sigma theta. Changes in stability class are important since the frequency distribution report categorizes wind speed and wind direction by stability class.

Stability Class		Description	Definition
1	A	Extremely Unstable	$22.5 \le \sigma_\theta$
2	B	Moderately Unstable	$17.5 \le \sigma_\theta < 22.5$
3	C	Slightly Unstable	$12.5 \le \sigma_\theta < 17.5$
4	D	Neutral	$7.5 \le \sigma_\theta < 12.5$
5	E	Slightly Stable	$3.8 \le \sigma_\theta < 7.5$
6	F	Moderately Stable	$2.1 \le \sigma_\theta < 3.8$
7	G	Extremely Stable	$\sigma_\theta < 2.1$

Analysis

One year of data was available for four wind sensors located on two towers. To get a quick look at the problem, the average stability class generated from the sigma theta measurements was computed for the 15-minute sigma thetas and the 60-minute sigma thetas. Intuitively, these numbers should be similar, but the 60-minute data consistently showed higher sigma theta values leading to lower stability classes (more unstable). For the 200 foot sensor, the 60-minute sigma theta indicated more unstable by one half stability class.

Tower & Height	15 Minute σ_θ Stability Class	60 Minute σ_θ Stability Class	Difference
Main 200'	4.48	3.99	.49
Main 100'	3.83	3.42	.41
Backup 90'	3.77	3.35	.42
Main 30'	2.82	2.52	.30

Then, for each hour of valid data, a 60-minute sigma theta was computed by finding the Root Sum Squared (RSS) average of the four 15-minute sigma theta values. The following table lists the arithmetic average of all 15-minute sigma thetas, the arithmetic average of the computed RSS sigma thetas representing the hour, and the arithmetic average of the 60-minute sigma thetas computed in the datalogger.

Tower & Height	15 Minute Sigma Thetas	RSS of Four 15 Minute Sigma Thetas	60 Minute Sigma Theta	Difference (60-min vs. RSS)
Main 200'	8.45	8.82	11.34	2.54
Main 100'	11.44	11.85	14.36	2.51
Backup 90'	12.74	13.21	15.71	2.50
Main 30'	17.01	17.43	19.78	2.35

The RSS average stability class was compared with the original 60-minute stability class. The following tables show the occurrences where using the 60-minute stability class produces a different stability class over the RSS computation of stability class. For example, for the main tower 200 foot elevation, using the 60-minute sigma theta and stability, 2442 observations (27.86%) were shifted one stability class to more unstable. 426 (4.86%) observations were 2 stability classes more unstable.

Tower & Height	Total Number	Stability Class Difference (Counts)							
		−3	−2	−1	0	1	2	3	4
Main 200'	8765		1	5	5804	2442	426	86	1
Main 100'	8765			7	6367	2021	319	51	
Backup 90'	8748		2	2	6405	1940	341	57	1
Main 30'	8765	1	2	7	7046	1594	112	3	

Tower & Height	Total Percent	Stability Class Difference (percent)							
		−3	−2	−1	0	1	2	3	4
Main 200'	100		.01	.06	66.22	27.86	4.86	.98	.01
Main 100'	100			.08	72.64	23.06	3.64	.58	
Backup 90'	100		.02	.02	73.22	22.18	3.90	.65	.01
Main 30'	100	.01	.02	.08	80.39	18.19	1.28	.03	

While the effect is reduced as elevation is reduced, approximately 30% of all observations are reported as one or more stability classes too unstable (when not RSS-ing the 15-minute values)The following table list the very few cases during the year when the sigma theta value computed by RSS-ing the 15-minute sigma thetas larger than the 60-minute value. In each case, the two values are very close near one of the class divisions, or one of the 15-minute values is higher than the rest, skewing the average up.

Main 200 foot

Date	Time	ST 15 min	ST 30 min	ST 45 min	ST 00 min	ST RSS	ST Diff	ST Hour	Stab Hour	Stab RSS	Diff Stab	Speed Wind	Diff Wind
12/24/2000	21:00	7.5	12.2	4.8	6.2	8.2	-2.5	5.7	E	D	-1	42.7	51.4
08/09/2000	07:00	10.5	9.8	7.6	5.2	8.5	-1.2	7.3	E	D	-1	27.5	14.7
08/27/2000	07:00	52.9	19.1	16.1	22.4	31.3	-8.9	22.4	B	A	-1	3.6	17.4
06/01/2000	10:00	6.1	3.8	3.7	2.3	4.2	-1.9	2.3	F	E	-1	9.5	10.1
08/30/2000	01:00	9.6	6.6	6.6	7.2	7.6	-0.2	7.4	E	D	-1	17.3	0.8
06/05/2000	14:00	31.5	12.7	11.3	8.6	18.4	-7.9	10.5	D	B	-2	12.9	51.9

Main 100 foot

Date	Time	ST 15 min	ST 30 min	ST 45 min	ST 00 min	ST RSS	ST Diff	ST Hour	Stab Hour	Stab RSS	Diff Stab	Speed Wind	Diff Wind
04/01/2000	15:00	46.1	18.3	15.5	11.9	26.7	-6.2	20.5	B	A	-1	3.9	14.8
12/24/2000	21:00	8.1	12.6	4.2	5.9	8.3	-3.1	5.2	E	D	-1	40.2	53.0
06/05/2000	14:00	32.7	19.1	15.9	16.0	22.0	-5.9	16.1	C	B	-1	10.0	71.0
07/15/2000	12:00	19.5	21.0	14.6	14.1	17.6	-1.9	15.7	C	B	-1	5.1	23.0
06/08/2000	12:00	18.6	17.8	18.9	14.8	17.6	-2.4	15.2	C	B	-1	12.7	8.9
09/08/2000	08:00	15.7	10.7	10.7	13.1	12.7	-0.9	11.8	D	C	-1	13.1	5.6
12/31/2000	18:00	12.9	12.7	12.5	12.3	12.6	-0.3	12.3	D	C	-1	2.0	155.7

Backup 90 foot

Date	Time	ST 15 min	ST 30 min	ST 45 min	ST 00 min	ST RSS	ST Diff	ST Hour	Stab Hour	Stab RSS	Diff Stab	Speed Wind	Diff Wind
12/31/2000	18:00	12.9	12.7	12.5	12.3	12.6	-0.3	12.3	D	C	-1	2.0	155.7
09/22/2000	09:00	18.1	11.6	10.8	11.0	13.2	-2.3	10.9	D	C	-1	13.5	7.3
06/05/2000	14:00	18.5	37.6	13.4	16.6	23.5	-7.2	16.3	C	A	-2	9.3	82.8
06/01/2000	10:00	27.6	6.7	14.3	4.9	16.1	-11.2	4.9	E	C	-2	10.5	16.2

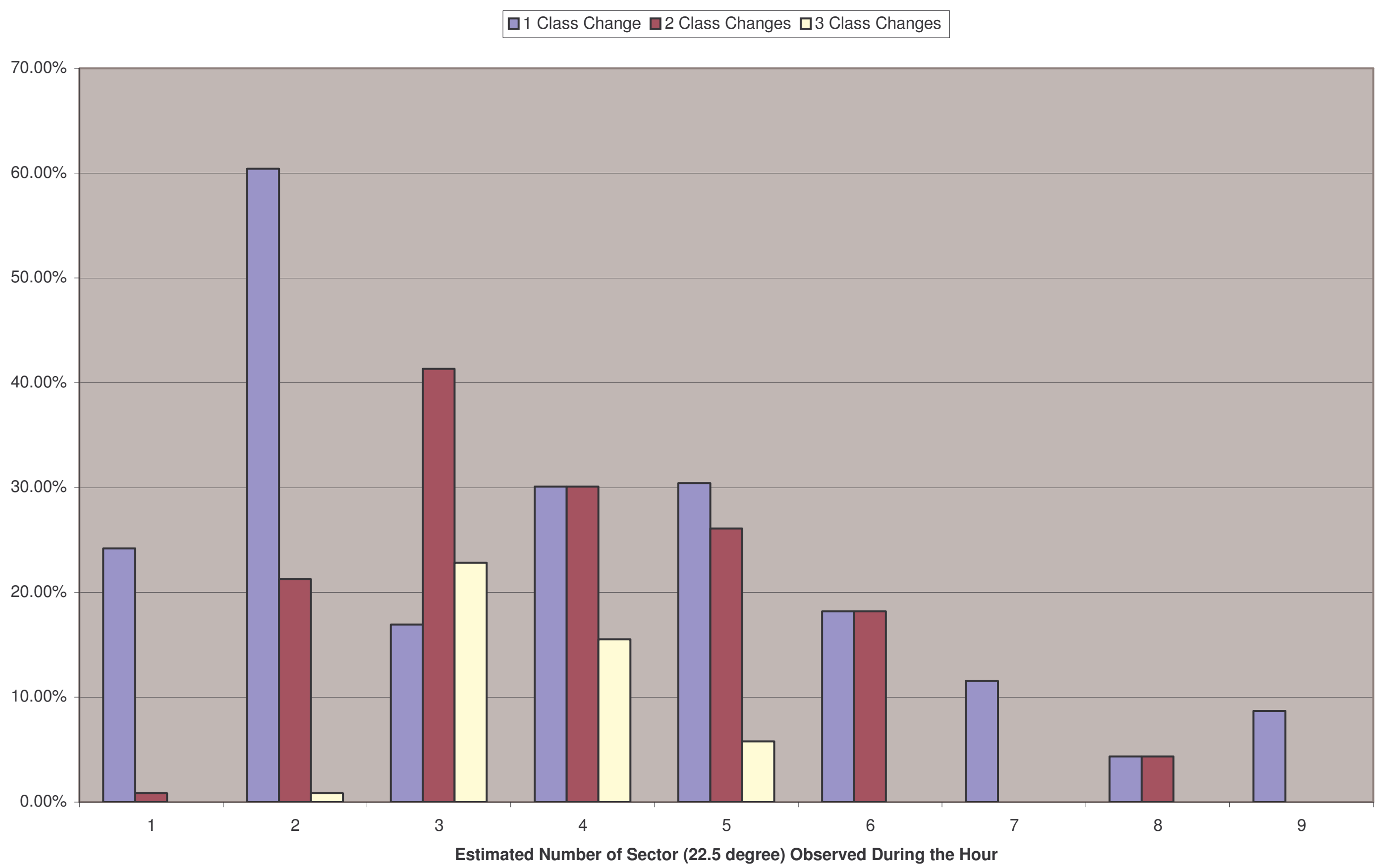
Main 30 foot

Date	Time	ST 15 min	ST 30 min	ST 45 min	ST 00 min	ST RSS	ST Diff	ST Hour	Stab Hour	Stab RSS	Diff Stab	Speed Wind	Diff Wind
12/24/2000	21:00	14.7	14.2	6.6	6.9	11.3	-4.5	6.8	E	D	-1	34.6	55.1
09/22/2000	10:00	19.2	20.4	22.1	14.3	19.2	-4.9	14.3	C	B	-1	6.2	14.8
09/23/2000	09:00	23.3	29.4	18.5	21.3	23.5	-3.2	20.3	B	A	-1	7.3	147.3
09/22/2000	09:00	27.7	43.9	18.5	21.3	29.5	-9.2	20.3	B	A	-1	7.3	20.9
02/03/2000	09:00	17.6	18.7	17.1	16.8	17.6	-0.8	16.8	C	B	-1	7.5	23.1
10/21/2000	13:00	42.5	12.0	9.1	5.3	22.7	-0.8	21.9	B	A	-1	12.2	6.8
12/31/2000	17:00	17.8	17.6	17.5	17.3	17.6	-0.3	17.3	C	B	-1	6.5	0.4
04/03/2000	08:00	30.5	24.7	20.5	14.8	23.3	-5.9	17.4	C	A	-2	2.3	37.6
06/01/2000	10:00	21.6	14.3	22.4	6.2	17.4	-11.2	6.2	E	C	-2	9.6	11.8
06/29/2000	13:00	65.8	26.3	8.5	4.7	35.8	-27.3	8.5	D	A	-3	11.6	64.8

The range of wind direction values during the hour was estimated from the 15 minute wind direction. This wind direction swing was then broken into sector size categories (22.5 ^{o}) intervals. The stability class difference was then grouped by these categories. The larger the change in wind direction, the more the 60-minute sigma theta values would be expected to differ from the RSS sigma theta values, but for hours when wind direction makes large changes, the sigma theta value tends to generate a stability class of A (extremely unstable) in both cases.

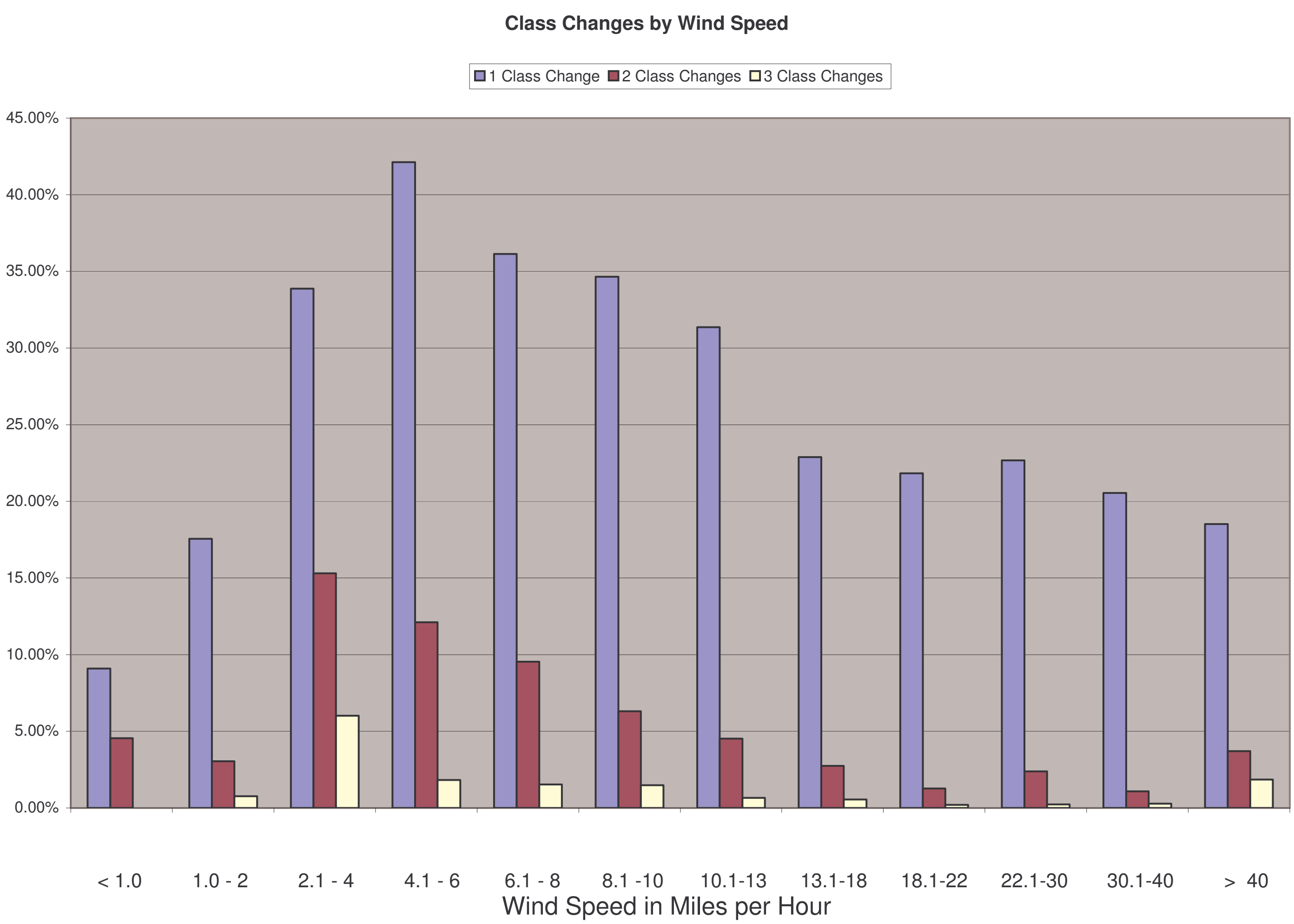
Wind Swing (Estimated) Sectors	Count	Stability Class Difference -2	-1	0	1	2	3	4
1	7256	0	4	5436	1756	60	0	0
2	955	0	0	167	577	203	8	0
3	254	1	1	45	43	105	58	1
4	103	0	0	25	31	31	16	0
5	69	0	0	26	21	18	4	0
6	44	0	0	28	8	8	0	0
7	26	0	0	23	3	0	0	0
8	23	0	0	21	1	1	0	0
9	23	0	0	21	2	0	0	0
10	6	0	0	6	0	0	0	0
11	6	0	0	6	0	0	0	0
12 & up	0	0	0	0	0	0	0	0
Totals	8765	1	5	5804	2442	426	86	1
		0.01%	0.06%	66.22%	27.86%	4.86%	0.98%	0.01%

Class Changes by Wind Direction Swings



The stability class difference was then grouped by twelve wind speed categories (one of the JFD report options). From the warning in 3.11, stability class differences might be expected for the low wind speed categories. However, the 60-minute sigma theta values differ from the RSS sigma theta values over the entire range of wind speed values.

Wind Speed (MPH) Category	Count	Stability Class Difference -2	-1	0	1	2	3	4
< 1.0	22	0	0	19	2	1	0	0
1.0 - 2	131	0	0	103	23	4	1	0
2.1 - 4	366	0	1	163	124	56	22	0
4.1 - 6	603	0	0	265	254	73	11	0
6.1 - 8	786	0	0	415	284	75	12	0
8.1 -10	808	0	1	464	280	51	12	0
10.1-13	1352	1	0	856	424	61	9	1
13.1-18	2369	0	1	1748	542	65	13	0
18.1-22	1026	0	0	787	224	13	2	0
22.1-30	878	0	1	655	199	21	2	0
30.1-40	370	0	0	289	76	4	1	0
> 40	54	0	1	40	10	2	1	0
Totals	8765	1	5	5804	2442	426	86	1
		0.01%	0.06%	66.22%	27.86%	4.86%	0.98%	0.01%



Conclusions and Recommendations

Re-Averaging and Editing Sigma Theta

As a result of this analysis, the re-averaging function will be changed to retain the 15-minute sigma thetas if no one minute data has been invalidated during the period and 60-minute sigma thetas will continue to be computed as the RSS of the 15-minute values.

For the one hour example shown at the beginning of the paper, the 15 and 60 minute sigma thetas were also computed from the 1-minute wind direction values using the Yamartino method. This sampling rate is too low for accurate results, but this method will be explored further and may be applicable in some cases.

Sample Time	Original Sigma Theta	Computed from 1-minute Sigma Theta	Computed from 1-minute Wind Direction
:15 15-minute	12.2	5.2	11.01
:30 15-minute	6.4	5.3	3.71
:45 15-minute	9.1	5.4	7.23
:00 15-minute	10.9	9.3	5.72
60-minute	19.0	6.6	17.90

Original Calculation of Sigma Theta

As for the original calculation of the 60-minute sigma theta, the suggestion in *ANSI/ANS-3.11-2000* is validated. This can be achieved most easily in the database or data management software, calculating the 60-minute sigma theta each hour from the 15-minute values. Dataloggers may also be modified to perform the RSS calculation.

If this becomes a topic of concern, advances in technology since the original papers cited in this paper may allow different algorithms to be considered in the dataloggers; either multi-pass methods or the use of some of the techniques considered by Fisher.

References

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